

On Simulating Type Ia Supernovae

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Abstract. Type Ia supernovae are bright stellar explosions distinguished by standardizable light curves that allow for their use as distance indicators for cosmological studies. Despite their highly successful use in this capacity, the progenitors of these events are incompletely understood. We describe simulating type Ia supernovae in the paradigm of a thermonuclear runaway occurring in a massive white dwarf star. We describe the multi-scale physical processes that realistic models must incorporate and the numerical models for these that we employ. In particular, we describe a flame-capturing scheme that addresses the problem of turbulent thermonuclear combustion on unresolved scales. We present the results of our study of the systematics of type Ia supernovae including trends in brightness following from properties of the host galaxy that agree with observations. We also present performance results from simulations on leadership-class architectures.

1. Introduction

1.1. Type Ia Supernovae

Type I supernovae are bright stellar explosions characterized by a lack of hydrogen in the observed spectrum. The type Ia sub-classification depends on the observation of a specific silicon line [1, 2]. These events are thought to be the result of a thermonuclear explosion consuming roughly one and a half solar masses of degenerate stellar material composed principally of C and O. The peak brightness of a type Ia supernova is set not by the explosion energy, but by the synthesis in the explosion of radioactive ^{56}Ni , which decays via the chain ^{56}Ni to ^{56}Co to ^{56}Fe releasing the observed energy. The vast majority of type Ia supernovae obey a correlation in which the peak brightness is positively correlated with the timescale over which the lightcurve decays from its maximum. This “brighter is broader” relation is known as the Phillips relation [3] and it allows the peak brightnesses to be calibrated so that these events may be treated as “standard candles” for determining distances. The correlation is understood

physically as stemming from having both the luminosity and opacity being set by the mass of ^{56}Ni synthesized in the explosion [4, 5, 6]. This relation has been exploited to make type Ia supernovae the premier distance indicators for cosmological studies.

While it is accepted that type Ia supernovae result from a thermonuclear runaway as described above, the progenitor systems of these events remain the subject of debate. Motivated largely by interest in cosmology, observational campaigns are gathering information about these events at an unprecedented rate. In particular, observations are finding a surprising range in the variation of the intrinsic brightness of these events, including very bright events that suggest more material burned than could originate from a single white dwarf [7, 8, 9, 10]. Contemporary research explores the efficacy of several progenitor systems for producing events with a range of brightnesses. The Phillips relation explains the first-order variations in peak brightness, but current research also aims to understand higher-order effects on the variation of the light curves and the physics behind the Phillips relation.

The progenitor system we assume is that of a single white dwarf composed principally of carbon and oxygen that has gained mass from a companion star [2]. The process of the white dwarf gaining mass from the companion is known as accretion. As the white dwarf gains mass, it is compressed and the temperature increases, which ignites thermonuclear reactions. Initially the reactions proceed relatively slowly, which drives convection in the core [11, 12]. As the temperature rises, the thermonuclear reactions proceed faster and faster, and eventually a subsonic flame is born that will eventually disrupt the star. Models that incorporate a transition from the subsonic deflagration to a supersonic detonation, allowing some expansion of the star prior to the detonation, best match some spectral features and the abundance stratification observed in the ejecta [13, 14, 15]. Our models assume this deflagration-to-detonation transition (DDT) paradigm.

1.2. Trends in Observations of Type Ia Supernovae

Many contemporary observations address correlations between the event and properties of the host galaxy such as its composition, age, and mass. Of particular interest are correlations between the brightness of an event and the isotopic composition and age, measured by the intensity of star formation, of the host galaxy. The proportion of material that has previously been processed in stars (i.e. elements other than hydrogen and helium, which are collectively referred to as “metals”) is a measurable property of the galaxy. The relative abundance of these elements is referred to as the galaxy’s “metallicity.” The presence of these elements in a progenitor white dwarf influences the outcome of the explosion by changing the path of nuclear burning, which influences the amount of ^{56}Ni synthesized in an event [16]. Because the decay of ^{56}Ni powers the light curve, metallicity can therefore directly influence the brightness of an event. Metallicity can also influence the outcome of an explosion in other ways, including changes in the structure of the white dwarf that result from metallicities influence on stellar evolution, sedimentation within the white dwarf, the nuclear flame speed, and additional sources of opacity [17].

Observational results to date are consistent with a shallow dependence of brightness on metallicity, with dimmer events in metal-rich galaxies, but are unable demonstrate a conclusive trend [18, 19, 20, 21]. Determining the metallicity dependence is challenging because the effect appears to be small, it is difficult to measure, and there are systematic effects associated with the mass-metallicity relationship within galaxies [19]. This effect is also difficult to decouple from the apparently stronger effect of the age of the parent stellar population on the mean brightness of type Ia supernovae [19, 21, 22]. Howell, et al. [21] note that the scatter in brightness of this observed relation is unlikely to be explained by the effect of metallicity.

The composition of a galaxy changes as stars form, consume hydrogen, and produce metals. Newly-formed galaxies are rich in hydrogen gas and undergo a period of intense star formation.

Observations target correlations between the brightness of an event and the age of a galaxy measured by the delay time (the elapsed time from the period of intense star formation). Some results indicate that the dependence of the Ia rate on delay time is best fit by a bimodal distribution with a prompt component less than 1 Gyr after star formation and a tardy component several Gyr later [23, 24]. Other studies only indicate a correlation between the delay time and the brightness of type Ia supernovae, with dimmer events occurring at longer delay times [19, 21, 20, 25].

2. Methodology

Our investigation into type Ia supernovae proceeds by isolating facets of the problem and subjecting these to study via statistically well-controlled ensembles of multidimensional simulations. We are trying to understand the physical mechanism of the explosion and isolating one effect at a time allows us to understand these processes. The parts of the problem we consider are properties of the progenitor white dwarf (such as structure and composition) that follow from properties the host galaxy (such as age and metallicity). The goal is to identify and quantify systematic trends in the brightness of events that follow from properties of the host galaxy. There are many possible systematic effects (outlined in [26]), and for each study we attempted to isolate all but one effect. Eventually, we hope to consider the interdependence of all of these effects in the construction of the full theoretical picture. Another feature of the type Ia supernova problem is that the explosion depends sensitively on the initial conditions, which are largely unknown.

We construct parametrized, hydrostatic massive white dwarf progenitor models with a variety of thermal and compositional structures thought to follow from the properties of the host galaxy and the accretion history [27]. The simulations are performed with a customized version of the FLASH simulation code [28, 29, 30] that includes a model flame and energetics scheme. We perform suites of simulations in which we vary properties of the progenitor white dwarf. The suites of simulations are controlled by a theoretical framework we developed that allows for statically well-controlled studies to determine any systematic effects due to properties of the progenitor that may follow from properties of the host galaxy.

Simulations with FLASH provide the bulk energetics of the explosion and an estimate of the yield of an event. We calculate detailed nucleosynthetic yields from the simulations by post-processing the thermodynamic histories of Lagrangian tracer particles embedded in the flow with a detailed nuclear network [31, 32].

In the subsections below, we highlight the flame model, sub-grid-scale turbulence and turbulence-flame interaction models, and the statistical framework we employ. Complete details of the methodology can be found in previously published results [31, 33, 34, 26, 32].

2.1. Model Flame and Energetics Scheme

The tremendous range in length between the white dwarf radius ($\sim 10^9$ cm) and the laminar nuclear flame width ($\approx [\kappa T/(\rho \varepsilon)]^{1/2} < 1$ cm for typical values of the thermal conductivity κ , density ρ , temperature T , and mass-specific heating rate ε), prohibits direct numerical simulation of type Ia supernovae. The problem is so severe that even an adaptive mesh simulation cannot resolve the actual diffusive flame front. For these reasons, simulations must rely on some sort of “model” flame. The structure of a one-dimensional laminar burning front is reasonably well understood [35], but the interaction of turbulence with this very thin flame front introduces small-scale structure in a way that is not well understood, even for flames in the laboratory. Capturing the net effect of this unresolved flame structure is necessary for a realistic model.

Thermonuclear flames in FLASH are tracked with an advection-diffusion-reaction scheme [36, 37, 34] that propagates an artificially broadened model flame. This model is based on the evolution of a reaction progress variable ϕ , where $\phi = 0$ indicates unburned fuel and $\phi = 1$

indicates burned ash, satisfying the advection-diffusion-reaction equation

$$\partial_t \phi + \mathbf{u} \cdot \nabla \phi = \kappa \nabla^2 \phi + \frac{1}{\tau} R(\phi). \quad (1)$$

Here $R(\phi)$ is a single chosen function, used everywhere, and κ and τ are parameters that are tuned locally so that the reaction front propagates at the physical speed of the real flame in that region [35] and is just wide enough to be resolved in our simulation. We use a modified version of the KPP reaction rate discussed by [37], in which $R(\phi) \propto (\phi - \epsilon)(1 - \phi + \epsilon)$, where $\epsilon \simeq 10^{-3}$, which is acoustically quiet and gives a unique flame speed [34].

In addition to the model flame, a realistic supernova model must accurately describe the energy release at the front and in the burned material, which is in a dynamic equilibrium. We performed a detailed study of the nuclear processes occurring in a flame in the interior of a white dwarf and developed an efficient and accurate scheme for numerical simulation [33, 34]. Tracking many nuclear species is computationally prohibitive, so simulations reproduce the energy release with an abstracted model with a tabulated state of the burned material produced from earlier directed simulations. This method accurately captures the thermal history of the material as it burns and evolves, which enables embedded particles to obtain accurate Lagrangian density and temperature histories. Detailed abundances can then be recovered by post-processing these time histories with a nuclear network including hundreds of nuclides. The nuclear processing can be well approximated as a three-stage process: Initially carbon is consumed, followed by oxygen, which creates a mixture of Si group and light elements that is in quasi-statistical equilibrium [38, 39] (also known as nuclear statistical quasi-equilibrium [40, 41, 42]); finally the Si group nuclei are converted to Fe group, reaching full NSE. In both of these equilibrium states, the capture and creation of light elements (via photodisintegration) is balanced, so that energy release can continue by changing the relative abundance of light (low nuclear binding energy) and heavy (high nuclear binding energy) nuclides, an action that releases energy as burned material rises and expands. Neutronization in the flame and dynamic ash, mainly via electron captures, is explicitly accounted for with tabulated rates from NSE calculations with 443 nuclides that include modern weak reaction rates [43]. Complete details of the NSE calculations may be found in [44] and the details of the implementation in our simulations may be found in [33, 34].

2.2. Sub-grid-scale Models

The model flame with its advection-diffusion-reaction scheme requires an input flame speed. For the one-dimensional laminar case, flame speeds are readily available from direct numerical simulations [45, 35]. In the multidimensional case, turbulence on unresolved scales boosts the flame speed. Realistic simulations must therefore include a method for determining the effect of this turbulence-flame interaction (TFI).

Initially we employed a method proposed by Khokhlov [36], that assumes perturbations to the flame front are dominated by the Rayleigh-Taylor instability. The method sets the flame front speed to $s = \max(s_\ell, 0.5\sqrt{\text{At} g m \Delta})$ where s_ℓ is the laminar flame speed, $\text{At} = (\rho_{\text{fuel}} - \rho_{\text{ash}})/(\rho_{\text{fuel}} + \rho_{\text{ash}})$ is the Atwood number, g is the local gravity, Δ is the grid resolution, and m is a calibrated constant. This prescription effectively keeps the flame speed above some lower limit that depends on the resolution of the simulation. [34]. Another approach that accounts for both unresolved Rayleigh-Taylor instability and background turbulence, originally proposed by Niemeyer and Hillebrandt [46] and developed in detail by Schmidt et al. [47, 48], relies on a dynamic measure of the local turbulent energy on sub-grid scales to enhance the local flame front propagation. The model sets the flame speed to $s = \sqrt{s_\ell^2 + C_t q^2}$, where q is a velocity that characterizes the sub-grid turbulence energy content and C_t is a constant taken to be 4/3 [49].

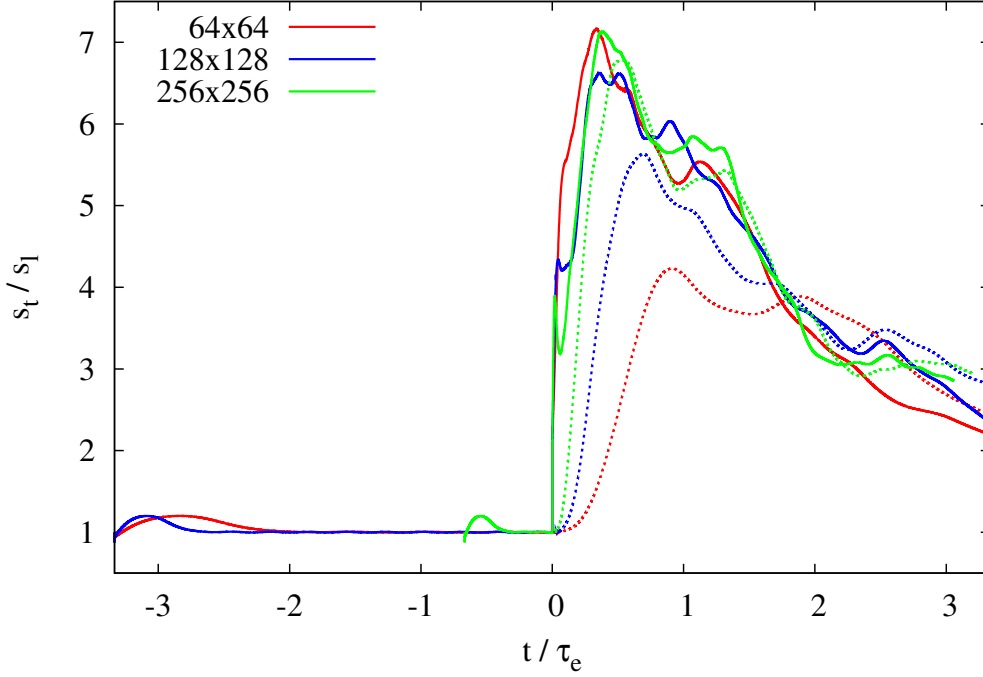


Figure 1. Convergence properties with resolution, with and without a TFI model. Shown is the ratio of the turbulent flame speed to the laminar flame speed as a function of the simulation time scaled by the turbulence eddy turnover time at $t = 0$, τ_e , in a channel with decaying turbulence. Three different resolutions are shown using a TFI model based on Charlette et al. [51] (solid lines) and without a TFI model (dotted lines). Convergence with resolution is achieved with the TFI model.

We have recently implemented a TFI model based on Colin et al. [50] and Charlette et al. [51] that reasonably predicts the behavior of turbulent flames in terrestrial experiments [52]. The method utilizes a local, instantaneous measure of the turbulence, in contrast to the dynamic turbulence model employed by Schmidt et al. [47]. In addition, the relation between the turbulent flame speed and the turbulent intensity is more carefully considered. Particular care has been taken to ensure that the turbulent flame speed predicted by the TFI model shows good convergence properties with resolution for the case of Kolmogorov turbulence. Figure 1 shows the ratio of the turbulent flame speed to the laminar flame speed for a flame propagating in a channel with decaying turbulence having an initial eddy turnover timescale τ_e . The ratio of the turbulent flame speed to the laminar flame speed is plotted as a function of simulation time scaled by τ_e . The solid lines show results from the TFI model following [51], while the dotted lines show results without the inclusion of a TFI model. Three tests were performed with cubic cells that cover the channel cross section with 64x64 cells (red), 128x128 cells (blue), and 256x256 cells (green) to demonstrate the necessity of a TFI model to obtain convergence with resolution.

2.3. Statistical Framework

The statistical framework we developed is designed to explore systematic trends in the brightness of type Ia supernovae. The ideas, however, are general and the same approach may be taken for other multi-physics applications. Our approach allows the evaluation of the average dependence

of properties of the model on underlying parameters by constructing a theoretical sample based on probabilistic initial conditions.

For our type Ia supernova research, we utilize two- and three-dimensional simulations in the DDT paradigm within the framework to evaluate the average dependence of the brightness of an event on parameters such as composition of the progenitor. The theoretical sample is based on probabilistic initial ignition conditions for the deflagration. Such sample-averaged dependencies are particularly important to the type Ia supernova problem because we seek to understand how models may explain features of the observed sample, particularly samples generated by large dark energy surveys utilizing type Ia supernovae as distance indicators.

The theoretical sample is constructed to represent statistical properties of the observed sample of type Ia supernovae such as the mean inferred ^{56}Ni yield and its variance. Within the DDT paradigm, the variance in ^{56}Ni yields can be explained by the development of fluid instabilities during the deflagration phase of the explosion. Choice of the initial configuration of the flame influences the growth of fluid instabilities, which result in varying amounts of ^{56}Ni synthesized during the explosion. While there are many sources of uncertainty, particularly in the progenitor white dwarf's composition and structure, variations of the initial configuration of the flame can be used to introduce fluid instabilities that produce the variance of ^{56}Ni yields seen in observations.

The physical initial conditions at ignition are not well known and most likely involve scales our simulations cannot resolve. Our initial conditions consist of an initially burned region that is resolved on the simulation grid. We found that perturbing a spherical flame surface (with radius $r_0 = 150$ km) with spherical harmonic modes (Y_l^m) between $12 \leq l \leq 16$ with random amplitudes (A) normally distributed between 0 – 15 km and, for three-dimensions, random phases (δ) uniformly distributed between $-\pi$ and π best characterized the mean inferred ^{56}Ni yield and sample variance from observations [26]. We write the perturbation as

$$r(\theta) = r_0 + \sum_{l=l_{\min}}^{l_{\max}} A_l Y_l(\theta) \quad (2)$$

$$r(\theta, \phi) = r_0 + \sum_{l=l_{\min}}^{l_{\max}} \sum_{m=-l}^l \frac{A_l^m e^{i\delta_l^m}}{\sqrt{2l+1}} Y_l^m(\theta, \phi). \quad (3)$$

With a suitable random-number generator, a sample population of progenitor WDs is constructed by defining the initial flame surface for a particular progenitor.

Our theoretical framework takes advantage of the fact that the outcome of an explosion is very sensitive to the initial conditions to produce the sample population. We note that other aspects of the problem such as the DDT density can significantly influence the ^{56}Ni yield as well. The mechanism by which a DDT occurs is poorly understood, but it might also contribute to the variation seen in the observations.

3. Performance Results

FLASH is a parallel, adaptive-mesh simulation code for multidimensional compressible reactive flows in astrophysical environments. At its heart is an explicit hydrodynamics method, and FLASH also includes solvers for the Poisson equation of self-gravity and the advection-diffusion-reaction scheme for propagating a model flame as described above. FLASH also provides an equation of state for a degenerate ionized plasma to describe stellar material. FLASH uses a customized version of the PARAMESH library [53, 54] to manage a block-structured adaptive grid, adding resolution elements in areas of complex flow. FLASH and PARAMESH use MPI for parallel communication.

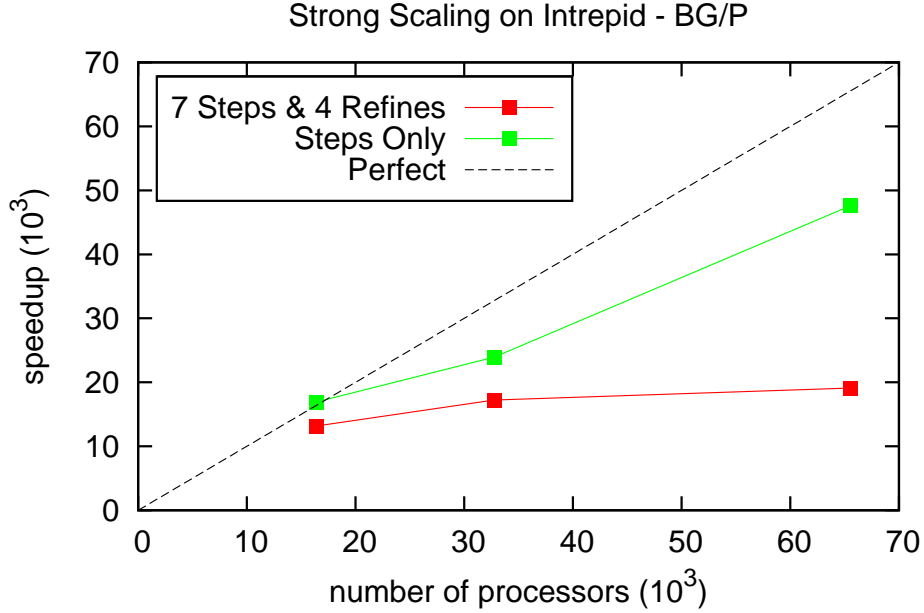


Figure 2. Strong scaling results for the ANL BG/P machine illustrating parallel speedup. Plotted is the speedup vs. number of processors for 16384, 32768, and 65536 processors. Also shown is the corresponding ideal scaling case.

FLASH has always demonstrated almost perfect weak scaling, winning the SC2000 Gordon Bell Prize (special category) [29], for scaling to 6420 processors of the Intel ASCI Red machine at LANL in 2000. As an example of strong scaling for production simulations, we present a performance test run on the BG/P machine at ANL. The results shown in Figure 2 are for a three-dimensional simulation under conditions typical for a production research run.

Simulations of type Ia supernovae utilize adaptive mesh refinement to track critical features of the simulation, particularly the location of the flame, at the highest resolutions. As the star explodes, the size of the simulation increases with time as the burned volume of the star increases. Accordingly, the proportion of simulation domain at the highest resolution grows, drastically increasing the size of the simulation. For this reason, the timing for the scaling study considered only a small part of the actual evolution. The test measured the amount of wall clock time taken to evolve a supernova simulation for a fixed period of simulation time (corresponding to 7 time steps, with mesh adaptations and load balancing every two time steps for a total of 4 adaptations). The study was run on 16384, 32768, and 65536 processors, and the simulation was for a late-stage of the deflagration, and is thus representative of a fully developed simulation. Figure 2 illustrates the parallel speedup,

$$\text{speedup} = \frac{16384t_{16384}}{t_N}$$

where t_N is the simulation time run on $N = 16384\text{--}65536$ processors. In this case, perfect scaling corresponds to the number of processors. Work is needed to improve the cost of mesh refinement.

4. Astrophysical Results

In Townsley et al. [26] we investigated the effect of the metallicity of the host galaxy by assuming it determined the initial neutron excess of the progenitor white dwarf. Weak interactions during nuclear burning, particularly electron capture, lead to the production of neutron-rich elements. The neutron excess of these elements is thought to drive the explosion yield toward stable iron-group elements. As a result, there is relatively less radioactive ^{56}Ni in the yield of iron-group elements, which results in a dimmer event [16]. Thus the introduction of neutron-rich metals into the progenitor white dwarf is thought to influence the brightness of an event by influencing the ^{56}Ni yield.

We investigated the role of metallicity by introducing ^{22}Ne into the progenitor white dwarf as a proxy for neutron-rich metals. The presence of ^{22}Ne influences the progenitor structure, the energy release of the burn, and the flame speed. The study was designed to measure how the ^{22}Ne content influences the competition between rising plumes and the expansion of the star, which determines the yield. We performed a suite of 20 two-dimensional DDT simulations varying only the initial ^{22}Ne in a progenitor model, and found a negligible effect on the pre-detonation expansion of the star and thus the yield of iron-group elements. The neutron excess sets the amount of material in NSE that favors stable iron-group elements over radioactive ^{56}Ni . Our results were consistent with earlier work calculating the direct modification of ^{56}Ni mass from initial neutron excess [16].

We expanded the study of Townsley et al. [26] to include the indirect effect of metallicity in the form of the ^{22}Ne mass fraction through its influence on the density at which the DDT takes place in Jackson et al. [27]. The study consisted of 30 “realizations” or sets of initial conditions (the randomized sample), and for each performed two-dimensional simulations with 5 transition densities between $1 - 3 \times 10^7 \text{ g cm}^{-3}$ for a total of 150 simulations. We found a quadratic dependence of the iron-group yield on the log of the transition density, which is determined by the competition between rising unstable plumes and stellar expansion. By then considering the effect of metallicity on the transition density, we found that the iron-group yield decreases slightly with metallicity, but that the ratio of the ^{56}Ni yield to the overall iron-group yield does not change significantly. Observations testing the dependence of the yield on metallicity remain somewhat ambiguous, but the dependence we found is comparable to that inferred from [55]. We also found that the scatter in the results increases with decreasing transition density, and we attribute this increase in scatter to the nonlinear behavior of the unstable rising plumes.

In Krueger et al. [22] we investigated the effect of central density on the explosion yield. We performed a suite of simulations from 30 realizations, and for each performed two-dimensional simulations with 5 central densities between $1 - 5 \times 10^9 \text{ g cm}^{-3}$. We found that the overall production of iron-group material did not change, but there was a definite trend of decreasing ^{56}Ni production with increasing progenitor central density (consistent with earlier studies [56, 57]). We attribute this result to higher rates of weak interactions (electron captures) that produce a higher proportion of neutronized material when the burning occurred at higher density. Similarly to the influence of metals, more neutronization means less symmetric nuclei like ^{56}Ni , and, accordingly, a dimmer event.

This result may explain the observed decrease in the brightness of events with increasing age measured as delay time from star formation. For the accreting white dwarf progenitor, only a narrow window in the range of possible accretion rates will produce a massive progenitor in which carbon can be centrally ignited, avoiding far off-center ignitions and subsequent gravitational collapse due to high electron-capture rates. Spherically symmetric models generally find central densities of $1.8 - 13.0 \times 10^9 \text{ g cm}^{-3}$, depending on the mass of the accreting WD [58, 59, 60]. This narrow window of rates implies a similar accretion duration for all progenitor systems.

Given that all progenitor systems seem to have the same duration of accretion, differences in progenitor age must then follow from differences in the time scale of evolution prior to the onset of

accretion. If there is a long period of cooling before the onset of mass transfer, the central density of the progenitor will be higher when the core reaches the carbon ignition temperature [61], thereby producing less ^{56}Ni and thus a dimmer event. Using the results of Lesaffre [61], we were able to relate our results to the progenitor age and compare our results to observations, shown in Figure 3. In addition, in this study we found considerable variation in the trends from some

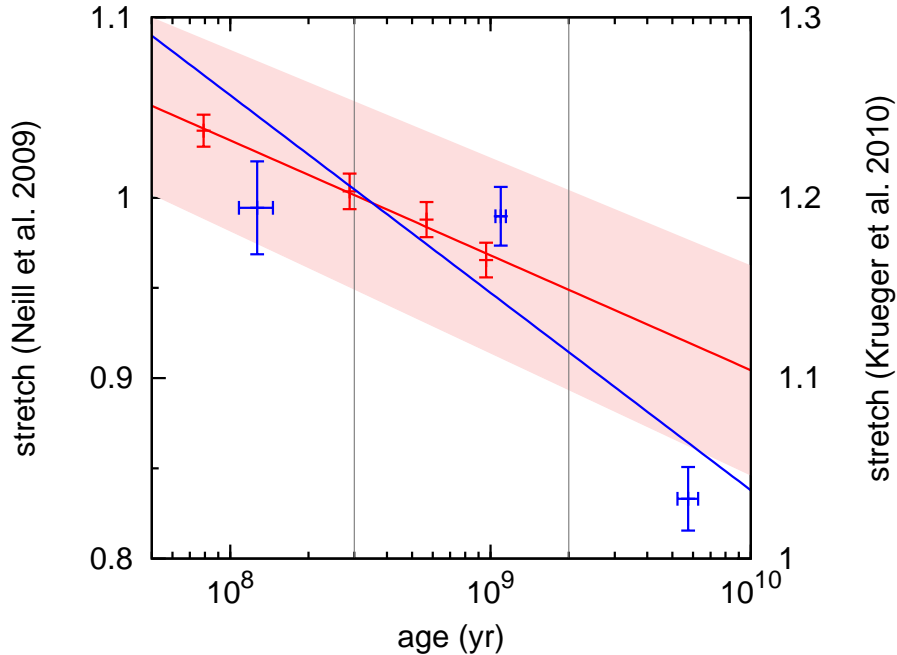


Figure 3. Plots of stretch (s) [21], a measure related to brightness [62], vs. age, comparing our trend in brightness with progenitor age to observational results. In red are results from our study, based on variations in ρ_c , along with the standard deviation (shaded region). In blue are the binned and averaged points from Figure 5 of [20], along with a best-fit line. Note the different scales: the overall offset to larger stretch in the simulations is due to the choice of DDT density. Adapted from [22].

realizations, stressing the importance of statistical studies [63].

5. Summary and Conclusions

We presented a snapshot of our research into the systematics of type Ia supernovae at the time of the Conference on Computational Physics held in Gatlinburg, TN, in the fall of 2011. We outlined our methodology and presented some highlights from our research into the systematics of type Ia supernovae. Our goal for this research it to develop sophisticated models that can reliably address issues like the intrinsic scatter of type Ia supernovae, an issue critical for the use of these events as cosmological distance indicators. Our contemporary research focuses on identifying trends in the brightness properties of the progenitor white dwarf that follow from the properties such as age and composition of the host galaxy. We developed a theoretical framework allowing study of the systematics of type Ia supernovae via statistically well-controlled suites of simulations [34], and we applied this method to studies of the composition and structure of the progenitor white dwarf.

Our results suggest that the direct effect of metallicity on the outcome of our multidimensional explosion models is very much in keeping with previous one-dimensional studies, e.g. [16]. A

suite of simulations showed a negligible effect on the expansion of the white dwarf prior to the detonation, and hence the yield [26]. We find that considering the indirect effect of metallicity on flame speeds and the deflagration to detonation transition produces a stronger trend. We found a quadratic dependence of the yield on the log of the transition density, which is determined by the competition between plume rise and stellar expansion during the deflagration phase. By considering the effect of metallicity on the transition density, we obtained a relationship between brightness and metallicity [27].

Our results from variations of the central density of the progenitor provide a theoretical explanation of the observed trend that type Ia supernovae from older host galaxies are systematically dimmer. Previous work [64, 65, 66, 67, 68] addressed the question of the impact of central density on brightness, with different studies reaching different conclusions. We found a strong trend of decreasing brightness with increasing central density due to increased rates of weak interactions that drive the burning toward more neutron rich products. By relating these results to the accretion history of the progenitor, we were able to obtain a relationship between brightness and age of the system [22].

A result drawn from all of the studies is dependence of the problem on the morphology of the deflagration. The deflagration phase of type Ia supernovae is strongly influenced by fluid instabilities and has a very nonlinear evolution. These results stress the necessity of a statistical study in order to capture the true trend.

Our conclusion is that we are beginning to understand the systematics of these events, a necessary step prior to addressing the issue of the intrinsic scatter. Future work will continue in this direction, with a study of the effect of the C/O ratio in the progenitor in progress. We also continue to refine and develop our models, particularly the sub-grid-scale turbulence and turbulence-flame interaction models. We have explored much of the parameter space of white dwarf progenitors and will perform targeted three-dimensional simulations to support our present results.

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